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On the significance of the external circuit, Langmuir and Bohm criterion for the stability of plasma fireballs

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This paper is devoted to studying the influence of the external circuit as well as the Langmuir and Bohm criterion on the stability of plasma fireballs. A simple mathematical model is suggested that describes why plasma fireballs can get unstable up to the point where they start pulsating. The predictions of this model are compared to measured experimental data. Furthermore, it is argued that the Bohm criterion in particular determines whether a stable plasma fireball can be formed. This adds to the current understanding that fireballs are preliminarily formed due to a change in the space charge in front of a positively biased electrode in surrounding plasma. It is argued that the space charge distribution near the vicinity of the anode surface might play a role but that the initial stages of fireball formation are dominantly driven by the requirement of the double layer to satisfy Bohm's sheath criterion and Langmuir's criterion. The same holds for a collapsing fireball. This paper shows that if the Langmuir and the Bohm criterion are not satisfied simultaneously, a fireball cannot reach a stable state and will start pulsating with a frequency that is proportional to the square root of the mass of the working gas ions.

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I. Introduction

When it comes to investigations on double layers, their stability is one major research topic. Over the decades a plethora of experimental and theoretical work has been carried out to enhance the understanding of stable and unstable double layers (DLs). DLs can be divided into strong [1, 2] and weak DLs [3-5], which both can become unstable. The main difference between these two is the ratio of the DL's potential drop and their thickness. Weak DLs exhibit a relatively small potential drop over a greater thickness while the potential drop within strong DL occurs over a comparably short length. The understanding of DL stability is of importance in various fields, such rf probes. This is also reflected in the the number of very recent review articles that have been published on the matter [6-8]. Other works investigate non-linear dynamics in different frequency regimes of DLs in magnetized and non-magnetized plasma [9-13]. These regions of opposing space charges also form the boundary of plasma fireballs (FBs). FBs are plasma regions with enhanced plasma density in front of a positively biased electrode that is immersed into existing background plasma. They are surrounded by a DL, which has a potential drop in the order of the first ionization potential of the working gas. Hence, the stability of DLs also has been extensively studied as part of FB research [14-16]. Most of the studied instabilities exhibit very high frequencies up to several hundred MHz. Normally, these instabilities introduce harmonics and nonlinearities into the DLs. However, in certain cases, (e. g. Ref. [16]), the DL can become so unstable that it collapses. After the collapse into a two-dimensional Debye sheath, the fireball re-ignites until it collapses again. This happens on times scales of several tens of microseconds and has been repeatedly observed in plasma fireball experiments. There is some basic consensus that this pulsing behavior is caused by an imbalance between electron and ion production and losses. Electron pairs and ions are produced inside a FB via electron impact ionization. The electrons are predominantly collected by the FB anode while the ions are expelled through the DL into the surrounding plasma. Baalrud et al. [8] established that a FB is formed when the condition $A_{anode}/A_W > \mu$ is satisfied. This relation between the anode surface area A_{anode}, the area through which ions are lost A_W and the ratio $\mu = \sqrt{2.3 m_e/m_i}$ determines not only if a DL is formed but also the shape of the resulting FB. A stable DL and, thus, FB can take on different shapes, namely spherical (i. e. a 'classical' FB), or elongated. For the latter configuration the term '*firerod*' has been coined [17, 18]. A firerod can evolve if the DL surface must be enhanced in order to compensate for the more rapid loss of electrons through the FB anode. If the expansion of the DL is sufficient for this necessary compensation, the FB, or firerod will be stable. This explains the shape and stability of such a plasma configuration to some extent. Nevertheless, to our knowledge, there is currently no model that accurately explains self-pulsating FBs (i. e. FBs that are not externally pulsed), let alone allows predictions about the temporal behavior of this absolute instability. This work proposes a simple physical model, which is based on the external circuit of the FB anode as well as the Langmuir and Bohm criterion for sheaths. It will be shown that the model describes self-pulsating FBs very well and has excellent agreement with experimental observations.

II. Physical Model

A typical FB circuit is shown in Fig. 1. It consists of an anode that is connected via a ballast resistor R to a DC power source. The FB plasma in front of the electrode is surrounded by a DL and produces two current densities through this sheath. The first is an electron current j_e, which penetrates the sheath and moves towards the anode. The primary electrons of the related current are accelerated up to kinetic energies by the potential drop through the DL. This allows additional ionization events inside the FB. The result of these ionizing collisions are an ion current with the current density j_i that is ejected through the DL outwards of the FB and another electron current density j_e'. The latter is collected by the anode and comprises of the primary electrons that were collected from the ambient background plasma as well as secondary electrons from the ionization events.

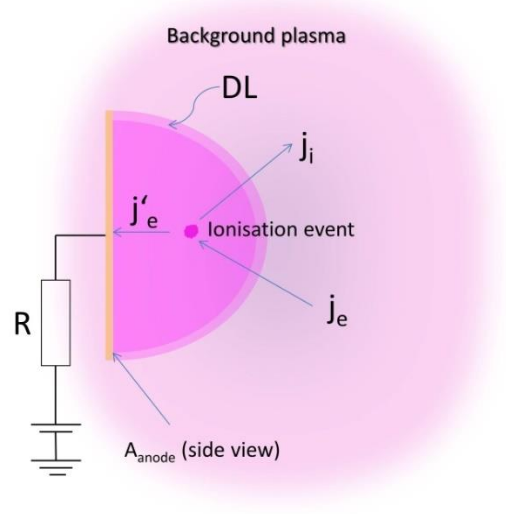

Figure 1: Schematics of a plasma FB setup with external circuit, electron and ion current densities and the surrounding double layer.

If there are only a few ionization processes in the FB, the current densities will differ only marginally and the FB can stabilize solely by expanding its surface. However, if the potential drop through the DL is sufficiently high, the number of additional ionizing collisions increases as well. In fact, this can become so extreme that the FB anode collects more current than the primary plasma source can deliver [16]. Generally speaking, a stable DL must fulfill the Langmuir criterion:

$$\frac{j_e}{j_i} = \left(\frac{m_i}{m_e}\right)^{1/2} \qquad (1)$$

On the other hand, the maximum current that can be collected by the FB anode is limited by the resistance of the electrode circuit and the applied voltage U:

$$I_{max} = \frac{U}{R} = j_e' \cdot A_{anode} \qquad (2)$$

Here A_{anode} denotes the surface area of the FB anode. j_e' is given by:

$$j_e' = e \cdot n_e \cdot v_e \qquad (3)$$

where n_e is the electron density inside the FB and v_e is the corresponding electron velocity. For example, argon plasma with single charged ions will yield a value of $(m_i/m_e)^{0.5}$ = (6.6 x 10-26/9.1 x $10^{-31})^{0.5}$ = 270. This indicates that the ratio of expelled ions and collected electrons must be 270. This value can be achieved rather easily for quasineutral plasma in which only a small fraction of electrons have high enough kinetic energy to induce ionization processes. However, it can be quite challenging to get this condition satisfied in an FB that ionizes efficiently, as will be discussed below.

The corresponding values for all the noble gases are given for convenience in the following Table 1:

Element	m_i [10^{-26} kg]	$\sqrt{m_i/m_e}$
Helium	6.6	85
Neon	33.4	191
Argon	66.4	270
Krypton	139.2	391
Xenon	218.0	489
Radon	368.6	636

Table 1: Square root of the ion and electron mass ratio for the noble gases.

The ions move roughly with the speed of sound inside the FB but leave the DL with a kinetic energy that is approximately the potential drop of the sheath [8, 19-21]. This was also experimentally verified by Stenzel et al. [16], who found the following values for the ion, and electron densities and drift velocities in a pulsating FB (note that these values were obtained in a self-pulsating FB and not in an externally pulsed one):

Parameter	symbol	value
Ion density inside the FB	n_i	4×10^{16}/m³
Electron density inside the FB	$n_e \sim 2\, n_i$	8×10^{16}/m³
Ion drift outside the FB	$v_{i,outside}$	8.4×10^3 m/s
Ion drift velocity inside the FB	$v_{i,inside}$	2×10^3 m/s
Electron drift velocity inside the FB boundary	$v_{e,iinside}$	2.3×10^6 m/s

Table 2: Measured ion and electron drift velocities and densities in an unstable FB in argon at 10^{-3} mbar. Note: n_e is deduced from the ion density.

The electron density is assumed to be about 2 x n_i because the FB size is normally in the order of the mean free path for electron impact ionization, or smaller and each ionization event produces one secondary electron. The electrode current connected to this FB I_{grid} was pulsating with a rise time of ~25 µs and a collapse time of ~ 50 µs. The temporal behavior of these parameters is depicted in the following Fig. 2:

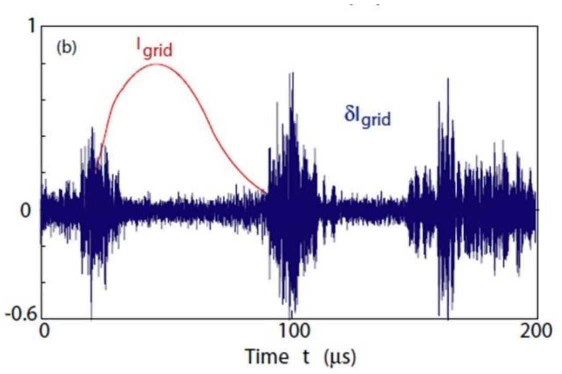

Figure 2: Current collected by a pulsating FB anode (red line), Taken from Ref. [15] with permission from © IOP Publishing. All rights reserved.

The experiments connected to the data in Fig. 2 were conducted in pure argon at around 10^{-3} mbar. Therefore, the densities and velocities inside the FB yield:

$$\frac{n_e}{n_i} \cdot \frac{v_e}{v_i} = 2\frac{v_e}{v_i} = 2 \cdot \frac{2.3 \times 10^6}{8.4 \times 10^3} = 540 \qquad (4)$$

As established before, this violates the Langmuir criterion, which would require the r.h.s. of Eq. (4) to be 270. Another aspect is the Bohm criterion that states that the ions must enter the DL at least with the velocity of sound, which they do as shown by the measurements. However, this is not enough to keep the ion current density high enough to maintain a stable sheath.

Block pointed out that in such a case the DL in the laboratory frame of reference starts moving with a velocity v_{DL} to meet the Langmuir and the Bohm criterion in its own frame of reference [22]. If $j_e' > j_i$, the DL will move in the direction of j_e' to compensate for this discrepancy. If, for example, Eq. (4) yields a value of 540, while the square root of the mass ratio should be 270, the velocity of a moving double layer can correct this discrepancy:

$$\frac{v_e - v_{DL}}{v_i + v_{DL}} = 270 \qquad (5)$$

Solving this yields v_{DL} = 1150 m/s. As the FB in the aforementioned experiments was about 6 cm in diameter, the collapse of the FB happens on a time scale $t_{coll} \approx s/v_{DL} = 6$ cm/1150 m/s = 5.2×10^{-5}s. These 52 μs are in excellent agreement with the measured 50 μs collapse time in the experiments by Stenzel et al. [15]. Another interpretation of this behavior is that the DL has to move because the plasma is not entirely quasineutral inside the FB. This can be seen from Eq. (4) because if $n_e = n_i$, the Langmuir criterion would be perfectly satisfied in that case. The velocity of the DL also corresponds to the ion transit time through the FB as was pointed out by Stenzel in Ref. [15]. Effectively, this means that in the frame of the moving DL v_i appears to be exactly twice as high as it is in the lab frame of reference and, thus, compensates for the higher electron collection rate by the FB anode.

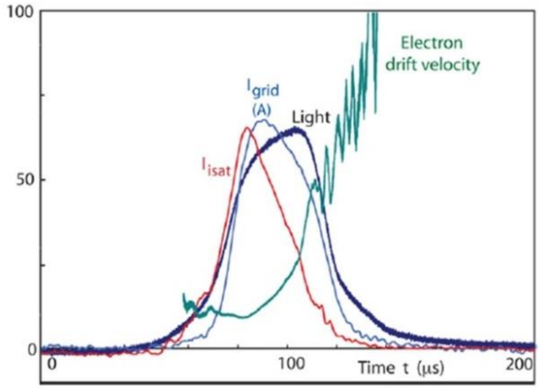

Figure 3: Current collected by a pulsating FB anode (blue line), light emission from the FB plasma (dark blue line), electron drift velocity (green line) and Langmuir probe saturation current Taken from Ref. [16] with permission from © IOP Publishing. All rights reserved.

Additional measurements in magnetized FBs by Stenzel [16] show that the plasma density decreases first, followed by the anode current and the emitted light, while during the regrowth all entities rise simultaneously. This is depicted in Fig. 3. The behavior of the plasma density is directly connected to the ion saturation current of a Langmuir probe, inserted into the FB plasma. The light, on the other hand was collected via a photodiode. It is reasonable that the light emission lasts longer than the elevated plasma density because the emitted light primarily stems from excited species in the FB, which are electrically neutral. Hence, they don't contribute to the ion saturation current or the FB anode current (denoted I_{grid} in Fig. 3). When the FB reignites, ionization and excitation increase at the same time and so does the light emission and the current on the anode and the Langmuir probe. This corroborates the explanation given in this paper. When the FB has fully evolved, the electrons are collected at a faster rate by the anode than the ions can be expelled through the surface. To fulfill the Langmuir and Bohm criterion, the DL has to start moving towards the anode. This hampers the ionization rates since the FB becomes smaller and smaller compared to the ionization mean free path. In fact, the FB diameter and the ionization mean free path λ_{mfp} are connected via [23, 24]:

$$D = \lambda_{mfp}\sqrt{\frac{T_e}{T_i}\frac{m_e}{m_i}} \qquad (6)$$

For $T_e \approx 1$ eV and $T_i \approx 0.01$ eV (roughly room temperature), the diameter of an argon FB is about $D = \lambda_{mfp} \times 4.3 \times 10^{-3}$ m. The ionization mean free path, on the other hand, can be expressed as:

$$\lambda_{mfp} = \frac{1}{\sigma \cdot n_n} \qquad (7)$$

Here, σ denotes the ionization cross section, which is in the order of 3×10^{-21} m² for electron energies between 15-20 eV [25] and n_n is the neutral gas density. The latter was calculated from the ideal gas law for a temperature of 300 K and a pressure of 10^{-3} mbar. This yields an ionization mean free path of 13.88 m, which indicates that only the electrons from the high energy tail of the EEDF can contribute to ionization processes. Plugging all these data into Eq. (6) results in a FB diameter of 6 cm, which is in perfect agreement with the observed, unstable FB as shown in the following Fig. 4.

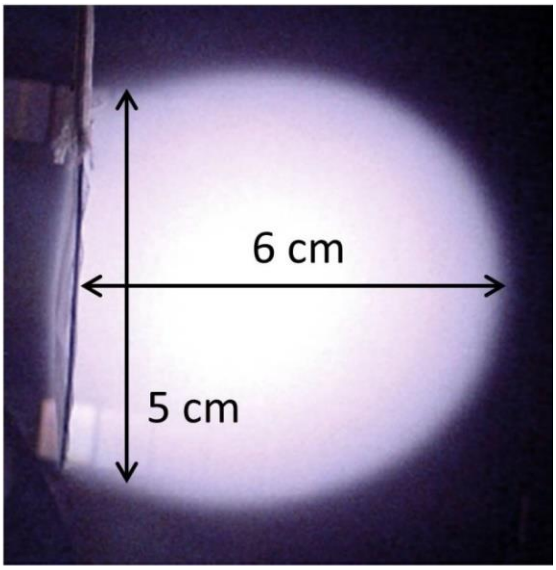

Figure 4: Photograph of a FB as obtained in the aforementioned experiments on a gridded anode (10^{-3} mbar, argon).

The collapse of the FB continues until a 2-D sheath is established that shields the anode potential against the rest of the plasma. This creates a relatively large potential drop within a thin DL and the probability of sheath ionization increases. Thus, the FB re-ignites. Hence, the increase in light emission happens at larger time scales compared to its collapse because initially the discrepancy between j_e' and j_i is small but becomes larger over time. However, the increase in the number of ionization events starts out relatively slowly because only a few highly energetic electrons can ionize neutrals at the beginning of the process. This process continues until the FB either reaches equilibrium or collapses again. Whether the FB becomes stable or unstable is also partially determined by the external circuit of the FB anode because the maximum velocity with which the electrons are collected by the anode is connected to the current density through the electrode surface:

$$v_{e,max} = \frac{j_{e'}}{e \cdot n_e} \qquad (8)$$

and

$$j_e' = \frac{I_{anode}}{A_{anode}} \qquad (9)$$

where I_{anode} denotes the current running through the FB electrode. Furthermore, this current is given by:

$$I_{anode} = \frac{U_{anode}}{R_{anode}} \qquad (10)$$

with the anode voltage U_{anode} and its resistance R_{anode}. Thus, one can obtain the maximal drift velocity of the electrons as a function of the FB anode's resistance, its surface and the electron density from Eqs. (8-10):

$$v_{e,max} = \frac{U_{anode}}{R_{anode}} \frac{1}{e \cdot n_e \cdot A_{anode}} \qquad (11)$$

For example, Stenzel et al. used a circular, gridded anode, which was 50 % transparent for the incoming electrons [16]. It had a radius of 2.5 cm. Hence, its electron collecting area was $A_{anode} = (2.5 \times 10^{-2}\ \mathrm{m})^2 \times \pi \times 0.5 = 9.8 \times 10^{-4}\ \mathrm{m}^2$. The anode was equipped with a 100 Ohm ballast resistor and biased to +50 V vs. ground, while the measured plasma density in Ref [16] was 10^{15} /m³ in absence of on FB. Hence, the maximum drift velocity with which the electrons can be collected by this anode is 3.2×10^6 m/s. However, in the absence of the FB, the bulk electrons move with their thermal velocity, which is given in Ref. [15] as 2 eV, or 9.1×10^5 m/s maximum. This bulk velocity is established because after the collapse of the FB into a 2-D sheath, the electron current to the FB anode is decreased substantially due to the Debye shielding and lack of ionization events. Such a decrease in the electron velocity and density would make it easier to fulfill the Langmuir criterion. On the other hand, it comes in conflict with Bohm's criterion. The reason is that the ion current through the sheath is very small. Only the energetic electrons from the high energy tail of the EEDF can induce ionization processes close to the sheath edge since only they have a large enough kinetic energy and ionization cross section. Thus, most of the ions will be created close to the anode sheath. Since the momentum transfer after an ionization impact by an electron is limited, the ions are born with roughly the thermal velocity of the neutrals close to the sheath edge:

$$v_{i,max} \cong v_{th,neutral} = \sqrt{\frac{2kT}{m_i}} \qquad (12)$$

In the case of argon at room temperature Eq. (12) yields 352 m/s.

As soon as the particles become charged ions, they are decelerated even further by the anode's electric field. Hence, the sound speed of the neutrals is roughly the maximum speed with which the ions can enter the sheath. Since the neutral sound speed is much smaller than the ion sound speed, Bohm's criterion is again severely violated. Therefore, the DL has to move in the direction away from the anode:

$$\frac{v_e + v_{DL}}{v_i - v_{DL}} = 270 \qquad (13)$$

With the electron drift velocity of 9.1×10^5 m/s and the ion velocity of 352 m/s, Eq. (13) yields v_{DL}=-3000 m/s for the velocity of the DL. The negative sign implies the direction of the movement of the DL (i.e. away from the anode in this case). Thus, the time in which the FB in this case expands to its original 6 cm diameter is: $t_{rise} \approx s/v_{DL} = 0.06\ \mathrm{m}/3000\ \mathrm{m/s} = 2 \times 10^{-5}$ s. This result is in very good agreement with the measured value of 25 µs from Ref. [15], which corroborates the suggested model.

III. Conclusions

In this paper a model based on the Bohm and Langmuir criterion is suggested to explain the underlying principles of self-pulsating FBs. The mechanisms that lead to this global instability have not been discussed so far. The proposed physical model corresponds very well with actual experimental observations. This model contributes not only to the understanding of the origins of self-pulsating FBs but also provides additional insights into the formation of stable FBs. The current understanding was based on the single assumption the an FB forms on the surface of a highly biased anode due to the change in the space charge structure in front of the electrode. However, the model outlined in this paper suggests that this view is incomplete. It can be argued that the Bohm criterion also plays a decisive role in the formation of a plasma FB. In order to keep this criterion valid and, at the same time, to fulfill Langmuir's criterion, the Debye sheath has to move away from the electrode surface and a FB is ignited. Whether the FB can reach a stable state or remains unstable, is dependent several factors. One factor is the ionization rate, which influences the electron density inside the FB. Another factor is discrepancy between the electron and ion drift velocities in the FB. If the ions cannot be expelled as fast as the electrons are collected, the DL will start moving to keep the Langmuir and Bohm criterion intact. To a certain extend the ion loss can be regulated by the FB via the ratio of FB surface to anode surface, as pointed out in Ref. [8]. However, if the right surface area ratio cannot be reached, the external circuit of the FB anode comes into play. If the maximum drift velocity of the electrons inside the FB plasma towards the anode is too high, the electrons are lost too fast and the FB will collapse. This collapse can be suppressed, either by increasing the anode area, the anode resistance, or both. The second option is to decrease the anode bias. However, this cannot be done at will because FB ignition requires an anode bias well above the ionization potential of the working gas.

On the other hand, for some applications or experiments, self-pulsating FBs might even be advantageous because they can create large fluxes of highly energetic ions. Such ion sources are of particular interest in surface modification technologies as well as space propulsion schemes (i.e. ion thrusters). Thus, the ability to steer the pulsation behavior of FBs might lead to interesting experiments and applications in the future.

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